HW 4 SOLUTIONS

SECTION 2.1

(2) $y_1 = \cos^2\theta$ $\qquad W(y_1, y_2) = y_1 y_2' - y_1' y_2$

$y_2 = 1 + \cos(2\theta)$

$y_1' = 2\cos\theta(-\sin\theta) = -2\sin\theta\cos\theta$

$y_2' = -2\sin(2\theta)$

$\Rightarrow W(y_1, y_2) = -2\sin(2\theta)\cdot\cos^2(\theta) + 2\sin(\theta)\cos(\theta)(1 + \cos(2\theta))$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$

$\Rightarrow W(y_1, y_2) = -2\sin\theta\cos\theta(1+\cos 2\theta) + 2\sin\theta\cos\theta(1+\cos(2\theta))$

$= 0$

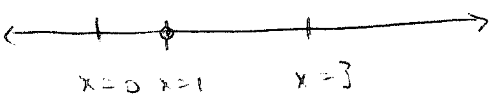
$\therefore$ $\boxed{\text{WRONSKIAN} = 0}$

(4) $(x-3)y'' + xy' + \ln|x| \cdot y = 0$, $y(1) = 0$, $y'(1) = 1$

$$\Rightarrow \quad y'' + \underbrace{\left(\frac{x}{x-3}\right)}_{=p(t)} y' + \underbrace{\left(\frac{\ln|x|}{x-3}\right)}_{=g(t)} y = 0.$$

$p(t)$ DISCONTINUOUS @ $x=3$

$g(t)$ DISCONTINUOUS @ $x=0$, $x=3$



$\therefore$ $\boxed{\text{A UNIQUE SOL}^{\underline{N}} \text{ WILL EXIST ON THE OPEN INTERVAL } (0,3)}$

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(6) $u = 2f - g$, $v = f + 2g$

$\Rightarrow$ $u' = 2f' - g'$, $v' = f' + 2g'$

$$W(u,v) = uv' - u'v = (2f-g)(f'+2g') - (2f'-g')(f+2g)$$

$$= 2ff' + 4fg' - gf' - 2gg' - 2ff' + 2gg' - 4f'g + g'f$$

$$= 4(fg' - f'g) + fg' - gf' = 4W(f,g) + W(f,g)$$

$$\therefore \boxed{W(u,v) = 5W(f,g)}$$

SECTION 2.2

(1) $6y'' - y' - y = 0$, CHARACTERISTIC EQ^{N}:

$$6r^2 - r - 1 = 0$$

$$(3r+1)(2r-1) = 0$$

$$\Rightarrow r = -1/3,\ 1/2$$

GENERAL SOLUTION:

$$\boxed{y(t) = c_1 e^{-t/3} + c_2 e^{t/2}}$$

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(2) $y'' + 5y' = 0$

$$r^2 + 5r = 0 \quad \Rightarrow \quad r(r+5) = 0$$

$$r = 0, -5$$

$$\therefore \quad \boxed{y(t) = c_1 + c_2 e^{-5t}}$$

(4)

$$y'' + y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

$$r^2 + r - 2 = 0$$

$$(r+2)(r-1) = 0$$

$$r = -2, 1$$

$$\Rightarrow y(t) = c_1 e^{t} + c_2 e^{-2t}$$

$$y(0) = c_1 + c_2 = 1$$

$$y'(t) = c_1 e^{t} - 2c_2 e^{-2t}$$

$$y'(0) = c_1 - 2c_2 = 1$$

$$\Rightarrow c_1 = 1, \quad c_2 = 0$$

$$\boxed{y = e^{t}}$$

$$\boxed{\text{As } t \to \infty, \ y(t) \to +\infty}$$

(8)

$$y'' - (2\alpha - 1) y' + \alpha(\alpha - 1) y = 0.$$

$$r^2 - (2\alpha - 1) r + \alpha(\alpha - 1) = 0.$$

$$\Rightarrow \quad r = \frac{(2\alpha - 1) \pm \sqrt{(2\alpha-1)^2 - (4)(1)\,\alpha(\alpha-1)}}{2}$$

$$= \frac{1}{2}\left[(2\alpha - 1) \pm \left(4\alpha^2 - 4\alpha + 1 - 4\alpha^2 + 4\alpha\right)^{1/2}\right]$$

$$= \frac{1}{2}\left[(2\alpha - 1) \pm 1\right]$$

$$= \alpha, \ \alpha - 1$$

$$\therefore \quad \boxed{y(t) = c_1 e^{\alpha t} + c_2 e^{(\alpha - 1) t}}$$

IF BOTH $\alpha > 0$ AND $\alpha - 1 > 0$, $|y(t)| \to \pm\infty$ AS $t \to \infty$

IF BOTH $\alpha < 0$ AND $\alpha - 1 < 0$, $y(t) \to 0$ AS $t \to \infty$.

$\therefore$

$$\boxed{\begin{array}{l} \alpha > 1 \Rightarrow y(t) \text{ IS UNBOUNDED AS } t \to \infty \\ \alpha < 0 \Rightarrow y(t) \to 0 \text{ AS } t \to \infty \end{array}}$$